MODELS OF NON-LINEAR DYNAMIC BEHAVIOUR OF CHAINS TRANSMISSION UNDER VARIABLE TECHNOLOGICAL LOADS

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ABSTRACT

Chains are machine components in cars, in geared systems and armored vehicles. The dynamics of chains typically are characterized by free motion and contact processes, which may include impacts and friction. Therefore, modeling by multi-body theory augmented by methods of contact mechanics represents an appropriate way to evaluate dynamic chain behavior. During its motion, due to the phenomena already mentioned appear parametric excitations, which induce a specific non-linear dynamic behavior. This paper gives some mathematical instruments to evaluate the non-linear chain dynamic behavior taking into account impacts, friction and parametric excitations.

1. INTRODUCTION

Typically, chains consist of chain elements that might be connected by ideal joints, joints with backlash, or journal bearing joints. Furthermore, and considering continuous variable transmission systems (CVT), chains may be connected by rocker pins or more complicated pin systems. Chains come into contact with guides and sprocket wheels, or they make contact with controllable sheaves. In the case of vehicles, chains make sprocket and ground contact. In all cases, contacts start by nullifying the relative distance between a guide, sprocket, or sheave and some chain element, and end by nullifying some force condition like normal force or static friction force. In between, contacts possess sliding friction or stick-slip transition, and local separation and closure might occur again. Altogether, all types of mechanical contact phenomena may occur between chains and their controlling elements.

Traditionally, chains have been modeled as a kind of uniform strings (Fritzer/4/, Yue/17/) because the methods available could hardly deal with chains composed of single elements. Nevertheless, polygonial effects were considered in Mahaligham/8/ and Chen and Freudenstein/2/. The basic work of Binder/1/ described the various possibilities of eigen frequencies and parametric excitations. In an approximative way, Naja and Marshek /9/ analyzed the contract forces for a steady-state behavior depending on the stress ratio between the tight and slack span. Nakanishi and Shabana/10/ developed very detailed chain models including contract forces for links, sprockets, and idlers with special application to large-scale tracked vehicles. A series of studies were performed that dealt with contract problems in multibody systems (Pfeiffer/11/, Prestl/13/, Seyfferth and Pfeiffer/15/, Glocker and Pfeiffer/5/, Pfeiffer and Glocker/13/). At University Politehnica of Bucharest some researches were conducted in the area at parametric vibrations of chains transmission (Bugaru and Motomancea, 1999).

2. CHAIN AND CHAIN ELEMENTS IN CONTACT DYNAMICS

2.1 General Remarks

In modeling chains and chain elements, we first must choose convenient coordinate frames. We start with an inertial coordinate base chosen in such a way that the geometry to the elements becomes as simple as possible; for example, located in the centers of sprockets or sheaves. We further fix at each element (chain element, sheave, sprocket, and guides) a moving body fixed coordinates system at a convenient point, for example, in the mass center. Sometimes it might be convenient to have a second body-related frame that is inertially fixed with one axis and rotates with a nominal speed, for example, for shafts and wheels. The system of coordinate frames must allow a complete and unambiguous geometrical description of all system elements.

Second, we must define a nominal motion that is nontrivial for chain dynamics. We start with a prescribed motion of the driving component, for example, one sprocket or one pulley. But we have to decide what a nominal of the chain system might look like. One could go back to the stringlike model as developed earlier, which is a reasonable basis. A more advanced nominal motion can be evaluated by taking into account the discrete chain structure and thus the polygonial effects and by not considering the detailed friction and impact phenomena. Experience shows that this approach seems to give good results and, moreover, a sound basis for more refined investigations.

2.2 Links and Joints

Typical links and joints for chains are very similar in different applications, such as roller chains in cars; rocker pin chains in CVT

gears, or even chains in tracked vehicles. The chain elements consist of a link that is connected to the neighboring link by a pin, a bushing, a rocker pin, or some similar object. In all, cases, these connections possess backlash that leads to relative motion. In roller chains or in CVT chains, the chain pins are lubricated by oil. They behave like a journal bearing and can be modeled as such. In many cases, it might even be sufficient to assume ideal joints being conveniently described by kinematical constraint equations. For heavy-tracked vehicles, the connections of the chain elements are not lubricated, and the rather large backlashes generate impulsive noise. We shall not consider this case here.
A typical chain element is shown in Figure1.

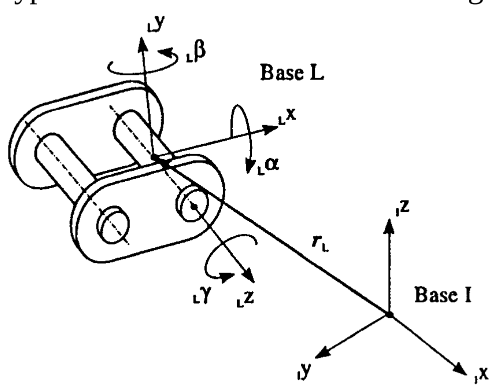


Figure 1. Typical chain link

In general, every element possesses six degrees of freedom, although in most applications it will be sufficient to consider only three degrees of freedom (two degrees-of-freedom translations, one degrees-of-freedom rotations).
From Figure 1, we get the coordinates of the link

$$_L\mathrm{r}=\begin{pmatrix} _Lx \\ _Ly \\ _Lz \end{pmatrix} \quad _L\varphi=\begin{pmatrix} _L\alpha \\ _L\beta \\ _L\gamma \end{pmatrix} \quad _L\mathrm{q}=\begin{pmatrix} _L\mathrm{r} \\ _L\varphi \end{pmatrix} \tag{1}$$

which of course might be written down in any coordinate frame. For the following roller chain and CVT models, we consider only the plane case ($_Lz$=0, $_L\alpha$=0, $_L\beta$=0). For the chain joints, we may either use an ideal joint model, which reduces further the number of degrees of freedom to one (namely, $_L\varphi$), and two additional geometrical constraints for $\boldsymbol{r}_L$. This leads to

$$_L\varphi=\begin{pmatrix} 0 \\ 0 \\ _L\gamma \end{pmatrix} \quad _L\mathrm{r}=\,_L\mathrm{r}=\,_L\mathrm{q}=\begin{pmatrix} _Lx \\ _Ly \\ 0 \end{pmatrix} \tag{2}$$

Alternatively, we may apply an approximation as a journal bearing with symmetric stiffness and damping matrices (Figure 2).

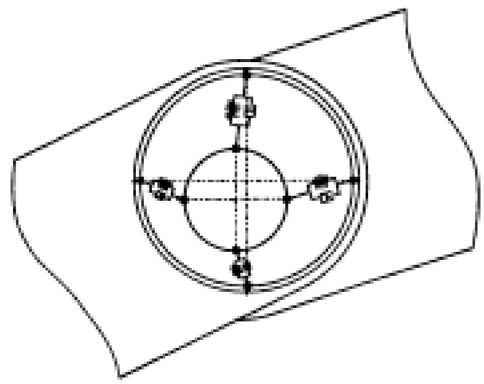


Figure 2. Chain connection as a journal bearing

2.3 Contact and Controlling Elements

CVT chains are controlled by movable and nonmovable sheaves; roller chains are controlled by sprockets and guides. The contact processes in the first case include impacts at the sheave entry and stick-slip phenomena during sheave contact. At the pulley exit, we might observe impulsive effects when the chain rocker pins leave the pulley. In the second case of roller chains, we observe not stick-slip processes but more sliding friction effects. Instead, we observe more impulsive processes on the guides and when entering or leaving the sprockets. Separation effects with subsequent contacts might occur along the guides. All effects are covered by the above theory. Nevertheless, we shall look into some details. Figure 3 illustrates a typical configuration of a CVT gear.

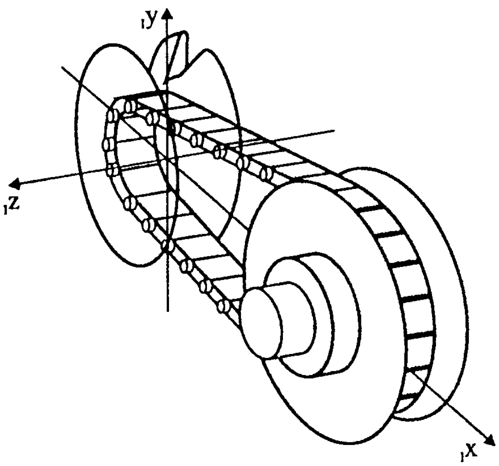


Figure 3. Typical CVT configuration

The chain elements may have contact with one pulley and are moving without such contacts in the strands between the two pulleys. Pulley 1 is the driving wheel. During motion, the rocker pin contacts with the pulley cones change. The rocker pin itself is modeled as one bolt with linear axial stiffness. We assume that Coulomb's law can be applied in the contact zone, and we know that the rocker pin moves within the conical pulley in two directions: radial and circumferential. This gives rise to a two-dimensional contact problem with nonlinear complementarity properties (Figure 4).

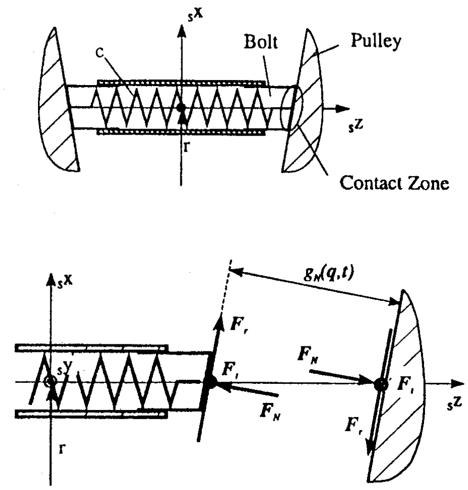


Figure 4. Rocker pin contacts

With respect to Coulomb's law, in this case, it turned out that computing time could be reduced significantly by approximating the dry friction characteristic by a continuous function. The relative normal distance $g_{Ni}(\boldsymbol{q}, t)$ and the relative tangential velocity $\dot{g}_{Ti}(q, \dot{q}, t)$ must be expressed in generalized coordinates and included in equation

$$M\ddot{q}-h-\left(W_N+W_G\mu_G \vdots W_H\right)\left(\lambda_N\ \lambda_H\right)=0\in\Re^f$$

$$\left(\ddot{g}_N\ \ddot{g}_H\right)=\left(W_N^T\ W_H^T\right)\ddot{q}+\left(\bar{w}_N\ \bar{w}_H\right)\in\Re^{n_N+n_H} \qquad (3)$$

where $\boldsymbol{M}\in\Re^{f,f}$ is the mass matrix, $\boldsymbol{q}\in\Re^f$ are the generalized coordinates, $\boldsymbol{h}\in\Re^f$ are all the forces except constraint forces, $g_N \in \Re^{n_N}$ is the vector of relative distances in n_N contacts, $W_N \in \Re^{f,n_N}, W_H \in \Re^{f,n_H}, W_G \in \Re^{f,n_N-n_H}$ are constraints matrices, $\bar{w}_N \in \Re^{n_N}, \bar{w}_H \in \Re^{n_H}$ are constraint vectors (mostly excitation terms), $\mu_G = diag\{-\mu_i \operatorname{sgn} \dot{g}_{Ti}\}$ is the sliding friction coefficient matrix, and $\lambda_N \in \Re^{n_N}, \lambda_H \in \Re^{n_H}$ are constraint forces (Lagrange vectors). The forces F_r, F_t in radial and circumferential directions depend on the constraint force vectors (λ_N, λ_H). Note that $g_{Ni}(\boldsymbol{q}, t)$ results from the approaching motion of the rocker pin as part of a strand element when coming nearer to the pulley.

The mechanical models for the pulleys are comparatively simple and very similar to those of sprockets. In both cases, we have wheels with three degrees of freedom in a plane model and six degrees of freedom in a spatial model. Stiffness and damping of the wheel bearings depend on their design, for example, roller or journal bearings (Figure 5):

$$ {}_W\mathrm{r}=\begin{pmatrix} {}_Wx \\ {}_Wy \\ {}_Wz \end{pmatrix} \quad {}_W\varphi=\begin{pmatrix} {}_W\alpha \\ {}_W\beta \\ {}_W\gamma \end{pmatrix} \quad {}_W\mathrm{q}=\begin{pmatrix} {}_W\mathrm{r} \\ {}_W\varphi \end{pmatrix} \tag{4} $$

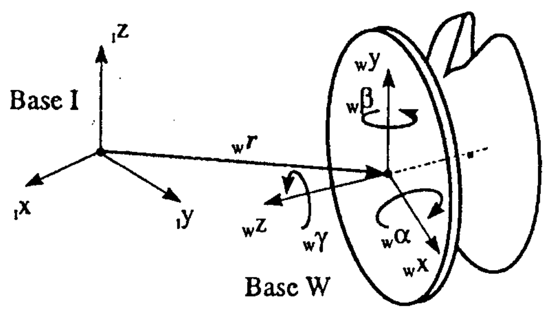


Figure 5. Wheel kinematics

where the index W stands for sprocket or sheave and the angles ${}_W\varphi$ of deviation are small. In the plane case, ${}_Wz=0$, ${}_W\alpha=0$, ${}_W\beta=0$.

The forces acting on a sheave are either external torques on the pulley shaft or internal forces coming from the contacts with the rocker pins. The latter is defined in Figure 4. Regarding a sprocket, again we have to consider the external torques from the camshafts and the rotational vibrations of the crankshaft. Besides the chain forces between the links, additional contact forces act on the chain bolts and on the sprocket toothing (Figure 6).

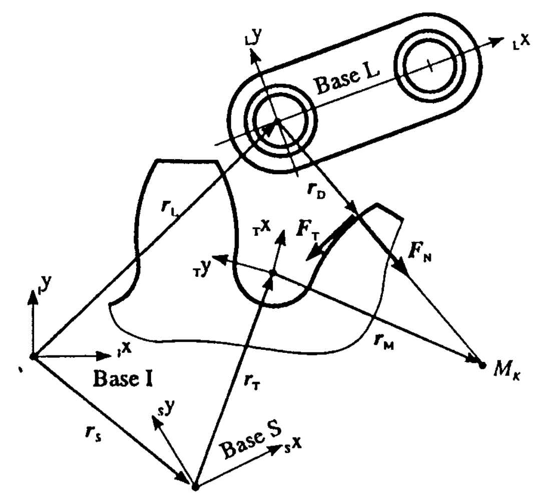


Figure 6. Contact forces between chain and sprocket

Particularly, the impulsive forces due to polygonial excitation, when a link enters the sprocket influence the sprocket dynamics and the dynamics of the chain strands.

Guides (Figure 7) are used in chain drives to reduce the vibrations of the chain strands and to preload the chain with a definite stress, which is produced by some tension device.

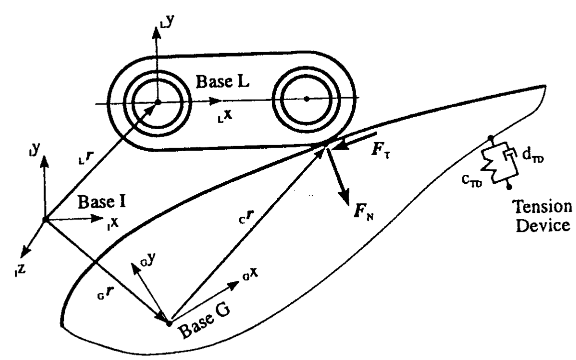


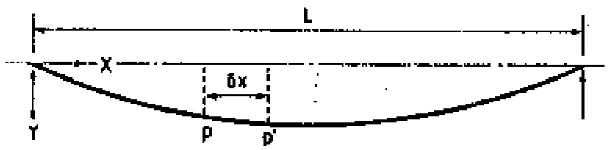
Figure 7. Guide chain dynamics

Therefore, guides develop their own dynamics so that the links might lose their contact with the guide. At each contact of the links, forces in normal and tangential directions act on the guide and the link plate. Again, we have to calculate the impulsive forces when a contact between a link and a guide becomes active.

3.PARAMETRIC VIBRATIONS OF CHAINS

One of the earliest studies in parametric vibrations of travelling power transmission devices, Mahalingam[8], showed that consideration of the chain as a heavy uniform string undergoing a longitudinal excitation admitted the possibility of a lateral, parametric, instability. The problem is shown diagrammatically in Figure 8.

Taking an *X*-*Y* coordinate system for the chain



deflection at P is y
deflection at P' is $y_{p'}$

Figure 8. Lateral deflection schematic for moving chain problem

and defining its longitudinal velocity by *V* we see that initially the deflection of some arbitrary point *P* along the unsupported length is y, and that this will change as the chain moves to another point, *P'*, through time δt.

The lateral deflection will now be given by

$$y_{p'} = y + \frac{\partial y}{\partial x}\delta x + \frac{\partial y}{\partial t}\delta t \qquad (5)$$

But since

$$V = \frac{\delta x}{\delta t} \qquad (6)$$

equation (5) becomes

$$y_{p'} = y + \left(V\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right)y\delta t \qquad (7)$$

Therefore the transverse velocity of *P'* is

$$V_{p'(trans)} = \left(V\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right)y \qquad (8)$$

Thus the transverse acceleration of *P'* is

$$a_{p'(trans)} = \left(V\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right)^2 y \qquad (9)$$

The equation of motion is

$$T\frac{\partial^2 y}{\partial x^2} = m a_{p'(trans)} \qquad (10)$$

where *T* is the chain tension.
Thus we have

$$\left(\frac{T}{m}\right)\frac{\partial^2 y}{\partial x^2} = V^2\frac{\partial^2 y}{\partial x^2} + 2V\frac{\partial^2 y}{\partial x\partial t} + \frac{\partial^2 y}{\partial t^2} \qquad (11)$$

(*T*/*m*) may be written as V_0^2, since V_0 may be defined as the velocity of transverse wave propagation, therefore we now have

$$V_0^2\frac{\partial^2 y}{\partial x^2} = V^2\frac{\partial^2 y}{\partial x^2} + 2V\frac{\partial^2 y}{\partial x\partial t} + \frac{\partial^2 y}{\partial t^2} \qquad (12)$$

A longitudinal excitation may be introduced in the form of a torsional oscillation of one of the sprockets. This can be expressed as $a\cos\omega t$ where ω is the frequency of torsional oscillation of the sprocket. The quantity *a* represents the periodic length change that the chain undergoes and so we can represent the instantaneous of the chain as $T+\Delta T\cos\omega t$. Substitution of this into equation (11) via our definition of *V*o leads to

$$(V_0^2 + \Delta V_0^2 \cos \omega t)\frac{\partial^2 y}{\partial x^2} = V^2 \frac{\partial^2 y}{\partial x^2} + + 2V\frac{\partial^2 y}{\partial x \partial t} + \frac{\partial^2 y}{\partial t^2} \qquad (13)$$

Because $y=y(x,t)$ we require a transformation into a coordinate with dependency solely on x or t. We choose to transform into $y_0 = y_0(t)$, where

$$y(x,y) = y_0(t)(\cos nx + i \sin nx) = y_0(t)e^{inx} \qquad (14)$$

where n can remain unspecified for the present. Substituting equation (13) into (14) enables the governing equation (now with time dependency only) to be rewritten

$$-(V_0^2 + \Delta V_0^2 \cos \omega t) y_0 n^2 e^{inx} = = -V^2 y_0 n^2 e^{inx} + 2V\dot{y}_0 n^2 e^{inx} + \ddot{y}_0 e^{inx} \qquad (15)$$

Canceling e^{inx} and rearranging gives

$$\ddot{y}_0 + i2Vn\dot{y}_0 + + \left[n^2\left(V_0^2 - V^2 + \Delta V_0^2 \cos \omega t\right)\right] y_0 = 0 \qquad (16)$$

Equation (16) is beginning to resemble the Mathieu form and in order to complete the analysis we make our further transformation whereby we eliminate the second term of equation (16). We do this by introducing a further variable in such a way that we have

$$y_0(t) = e^{-iVnz} u(z) \qquad (17)$$

Argument ωt is restated also

$$\omega t = 2z \qquad (18)$$

Differentiation of equation (17) with respect to time so that we have

$$\ddot{y}_0 = \frac{dy_0}{dz}\frac{dz}{dt}; \; \ddot{y}_0 = \frac{d^2 y_0}{dz^2}\left(\frac{dz}{dt}\right)^2$$

enables the following substitutions to be made in equation (16)

$$\left[-V^2 n^2 u(z) - i2Vnu'(z) + u''(z)\right]\left(\frac{dz}{dt}\right)^2 + + i2Vn\left[-iVnu(z) + u'(z)\right]\left(\frac{dz}{dt}\right) + + [n^2 V_0^2 u(z) - n^2 V^2 u(z) + n^2 \Delta V_0^2 \cos 2z u(z)] = 0 \qquad (19)$$

where the primes denote differentiation with respect to z.

If we stipulate that $z = t$, then $\omega = 2$, and we can revert to the dot derivative notation; expanding equation (19) gives

$$\ddot{u} + \left(n^2 V_0^2 + n^2 \Delta V_0^2 \cos 2t\right)u = 0 \qquad (20)$$

We can simplify further by stating

$$\omega_0^2 = V_0^2; \quad \varepsilon = \Delta V_0^2 \qquad (21)$$

which leads to

$$\ddot{u} + \left(n^2 \omega_0^2 + n^2 \varepsilon \cos 2t\right)u = 0 \qquad (22)$$

This is a linear undamped Mathieu-Hill equation for which we can easily derive the stability boundary for principal parametric resonance.

Therefore we arrive at the following

$$\frac{1}{n} = \omega_0 \pm \frac{\varepsilon}{4\omega_0} \qquad (23)$$

This is shown in Figure 9 (note that

$\omega = 2\omega_0$ for this principal parametric resonance case). The excitation is provided by ε and we can relate this back to chain tension since

$$\varepsilon = \frac{\Delta T}{M} \tag{24}$$

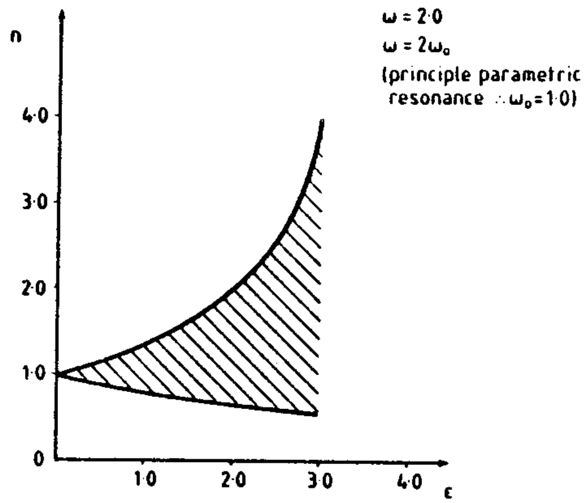


Figure 9. Stability chart showing the transition curve and unstable zone for principal parametric resonance in the moving chain problem

Therefore the larger the torsinal oscillation of the sprocket the larger the magnitude of ΔT, and hence ε, and the wider the region of instability at principal parametric resonance. Substitution of ε = 1, 2, 3, into equation (22) enables the instability region for $\omega = 2\omega_0$; ω = 2 to be portrayed (see Figure 9).

4. CONCLUDING REMARKS

The non-linear dynamics behaviour of chains under variable tehnological loads were investigated by applying multibody theory in connection with contact mechanical properties and parametric excitations.

The problem of multiple and nondecoupled contacts of chains offers a special treatment of multibody systems, including aspects of non-linear complementarity theories of parametric excitations. Such tools are an indispensable prerequisite for modeling complex systems like chains transmission under variable tehnological loads with, their numerous impulsive and friction-induced contact processes.

Further research should be invested in measurements, numerical and computer time problems.

REFERENCES

1. BINDER, R.C., MECHANICS OF THE ROLLER CHAIN DRIVE, Prentice Hall, Englewood Cliffs, NJ,1956.
2. CHEN, C.K. and FREUDENSTEIN, F., "TOWARDS A MORE EXACT KINEMATICS OF ROLLER CHAIN DRIVES", *ASME, J Mechan., Trans., Auto. Des.* **110**, pp. 169-275, 1988.
3. FRITZ, P. and PFEIFFER, F., "DYNAMICS OF HIGH SPEED ROLLER CHAIN DRIVES", *Proceedings of the 15th Biennial Conference on Mechanical Vibrations and Noise*, Boston, MA, September 17-21, 1995.
4. FRITZER, A.,"NICHTLINEARE DYNAMIK VON STEUERTRIEBEN", VDI-Fortschrittsberichte, Reihe 11, Nr. 176, VDI-Verlang, Düsseldorf, 1992.
5. GLOCKER, C. and PFEIFFER, F., "COMPLEMENTARITY PROBLEMS IN MULTIBODY SYSTEMS WITH PLANAR FRICTION", *Arch. Appl. Mech.* **63**, pp. 452-463, 1993.
6. GLOCKER, C. and PFEIFFER, F., "MULTIPLE IMPACTS WITH FRICTION IN RIGID MULTIBODY SYSTEMS", *Nonlinear Dynamics*, **7**, pp. 471-497, 1995.
7. LEMKE, C.E., "RECENT RESULTS ON COMPLEMENTARITY PROBLEMS", in *Nonlinear Programming*, J.B. Rosen et al., eds., Academic Press, New York, pp. 349-384, 1970.
8. MAHALIGHAM, S., "POLYGONIAL ACTION IN CHAIN DRIVES", *J. Franklin Inst.* **256**(1), pp. 23-28, 1958.
9. NAJA, M.R. and MARSHEK, K.M., "ANALYSIS OF THE SPROCKET LOAD DISTRIBUTION", *Mechan. Machine Theory* **18**(5), pp. 349-356, 1983.
10. NAKANISHI, T. and SHABANA, A.A., "CONTACT FORCES IN THE NONLINEAR DYNAMIC ANALYSIS OF TRACKED VEHICLES", *Int. J. Numerical Methods Eng.* **37**, pp. 1251-1275, 1994.
11. PFEIFFER, F., "MECHANISCHE SYSTEME MIT UNSTETIGEN UBERGANGEN", *Ingenieur-Archiv* **54**, pp. 232-240, 1984.
12. PFEIFFER, F., "COMPLEMENTARITY PROBLEMS OF STICK-SLIP VIBRATIONS, DYNAMICS AND VIBRATION OF TIME-VARYING SYSTEMS AND STRUCTURE", *ASME* **56**, pp. 43-50, 1993.
13. PFEIFFER, F. and GLOCKER, C., *MULTIBODY DYNAMICS WITH UNILATERAL CONTACTS*, John Wiley, New York, 1996.
14. PRESTL, W., "ZAHNHÄMMERN IN RÄDERTRIEBEN VON DIESELMOTOREN", *VDI Fortschrittsberichte* Reihe 11, Nr. 145, VDI-Verlang, Düsseldorf, 1991.
15. SEYFFERTH, W. and PFEIFFER, F., "DYNAMICS OF RIGID AND FLEXIBLE PART MATING WITH A MANIPULATOR", in *Proceedings of the IMACS-IFAC Sympositum*, Lille, 1991, pp. 419-424.
16. SRNIK, J. and PFEIFFER, F., "STIMULATION OF A CVT CHAIN DRIVE AS A MULTIBODY SYSTEM WITH VARIANT STRUCTURE", *CISS-First Conference of International Simulation Societies*, Zürich, August 22-25, 1994.
17. YUE, G., "BELT VIBRATION CONSIDERATION MOVING CONTACT AND PARAMETRIC EXCITATION", in *Proceedings of the International Power Transmission and Gearing Conference* **1**, pp. 311-318, 1992.